Exam 2 Review
October 17, 2011

1. Sketch the level curves of the function $g(x, y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$
2. Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$ for only positive values of $x, y, f(xy)$.
3. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.
4. Evaluate $\lim_{(x,y)\to(1,2)}(x^2y^3 - x^3y^2 + 3x + 2y)$ and justify your solution.
5. Where is the function $f(x, y) = \frac{x^2-y^2}{x^2+y^2}$ continuous?
6. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y.
7. If $f(x, y) = \sin\left(\frac{x}{1+y}\right)$, calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
8. Calculate f_{xxyz} if $f(x, y, z) = \sin(3x + yz)$.
9. Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there. Then use the linearization to approximate $f(1.1, -0.1)$.
10. If $z = f(x, y) = x^2 + 3xy - y^2$ find the differential dz.
11. If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2e^{-t}$, and $z = r^2s\sin t$ find the value of $\frac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$.
12. Find $y^{'}$ if $x^3 + y^3 = 6xy$.
13. Find the directional derivative $D_uf(x, y)$ if $f(x, y) = x^3 - 3xy + 4y^2$ and u is the unit vector given by angle $\theta = \frac{\pi}{6}$. What is $D_uf(1, 2)$?
14. If $f(x, y) = xe^y$, find the rate of change of f at the point $P(2, 0)$ in the direction from P to $Q(1/2, 2)$.
15. Find the maximum rate of change of $f(x, y) = \frac{y^2}{x}$ at the point $(2, 4)$ and the direction in which the maximum occurs.
16. Find the equation of the tangent line and the normal line to the surface $2(x - 2)^2 + (y - 1)^2 + (z - 3)^2 = 10$ at the point $(3, 3, 5)$.
17. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.
18. Find the dimensions of the rectangular box with largest volume if the total surface area is given as $64cm^3$.
19. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant c.